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EFFECT OF INTERNAL RADIATION ON THE TEMPERATURE DISTRIBUTION OF A SPHERICAL SHELL

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NOTICES

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FOREWORD

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ABSTRACT

This report is a continuation of a study of the effects of internal heat transfer on the temperature of hollow spacecraft and the requirements for thermal modeling. Considered herein is the effect of internal heat transfer by radiation on the temperature distribution. The equation governing the heat transfer of a spherical shell exposed to parallel radiation is derived; conduction and radiation are considered. The general equation is simplified by assuming steady state, and a numerical method is given to solve the steady state equation. A computer program is described which employs the method. Solutions of the steady state equation are graphically presented and discussed. The requirements for temperature preservation in thermal modeling are derived. The possibility of thermal modeling without temperature preservation is discussed. It is observed that for an inside emissivity to outside emissivity ratio greater than one, the requirement for duplication of the other dimensionless ratio can be relaxed.

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NOMENCLATURE

A^k	Conducting area
A^p	Projected area
A^R	Radiating area
a	Radius of sphere
b	Shell thickness
c	Specific heat capacity
D	Density
E	Ratio of emissivities
F(x)	Parallel radiation function
$F_{\theta-\theta'}$	Form factor from area at θ to area at θ'
$f(\theta)$	Parallel radiation function
$G_{\theta-\theta'}$	Form-surface factor from area at θ to area at θ'
H(x)	Step function to limit F(x)
$h(\theta)$	Step function to limit $f(\theta)$
k	Thermal conductivity
N_c	Dimensionless ratio
N_R	Dimensionless ratio
N_s	Dimensionless ratio
p_1	Rate of heat transfer into sector by conduction
p_2	Rate of heat transfer out of sector by conduction
p_3	Rate of heat transfer into sector from source
p_4	Rate of heat transfer out of sector by radiation
p_5	Rate of heat transfer into sector by radiation
p_{in}	Net rate of heat transfer into sector
$p_s(r)$	Parallel radiation function
q_s	Area average of p_s
r	Distance from axis of symmetry
T	Temperature
T_m	Arbitrary temperature

t	Time
t_m	Arbitrary time
V	Volume
x	Cosine of θ
Δx	Step size
y	Dimensionless time
Z	Dimensionless temperature
Z_1	Lower bound of $Z(-1)$
Z_o	Trial value of $Z(-1)$
Z_u	Upper bound of $Z(-1)$
α_i	Inside absorptivity
α_s	Absorptivity of radiation from source
ϵ_i	Inside emissivity
ϵ_o	Outside emissivity
θ	Angle from axis of symmetry
ρ_i	Inside reflectivity
σ	Stefan-Boltzmann constant

SECTION I
INTRODUCTION

This report is a continuation of a study of the effects of internal heat transfer on the temperature of hollow spacecraft and the requirements for thermal modeling. Considered herein is the effect of internal radiative heat transfer on the temperature distribution.

In Ref. 1, the effect of internal convection on the temperature of a spacecraft model of arbitrary shape, subjected to parallel radiation, was considered, and the transient temperatures were calculated. With the aid of numerical results, the conditions under which convection can be neglected were determined. Also, thermal modeling rules were derived for testing scale models. Thermal modeling is a valuable technique in ground testing of spacecraft. Several aspects of thermal model testing in space simulation chambers are discussed in Ref. 2.

In this report, the geometry is restricted to a spherical shell, and solutions are obtained for the steady state case. The spherical shell is subjected to parallel radiation which is considered uniform for the calculations presented herein; however, the equations derived and the numerical method allow axially symmetric nonuniformities. These somewhat arbitrary restrictions were imposed so that relatively simple calculations resulted, which nevertheless bring out features of a more general nature. By carrying out selected calculations of this type, it is hoped that better insight can be obtained concerning the importance of various parameters to thermal testing in space simulation chambers.

SECTION II
MATHEMATICAL ANALYSES

2.1 GENERAL EQUATIONS

The system to be considered is shown in Fig. 1 with vacuum inside and outside of the sphere. It is assumed that the shell thickness, b, is small enough to disallow any temperature gradient in the radial direction. It is also assumed that the inside of the sphere emits and reflects diffusely. The parallel radiation, p_s, is allowed to be a function of r. This may be useful at a later time in investigating the effects of nonuniformities of solar simulators.

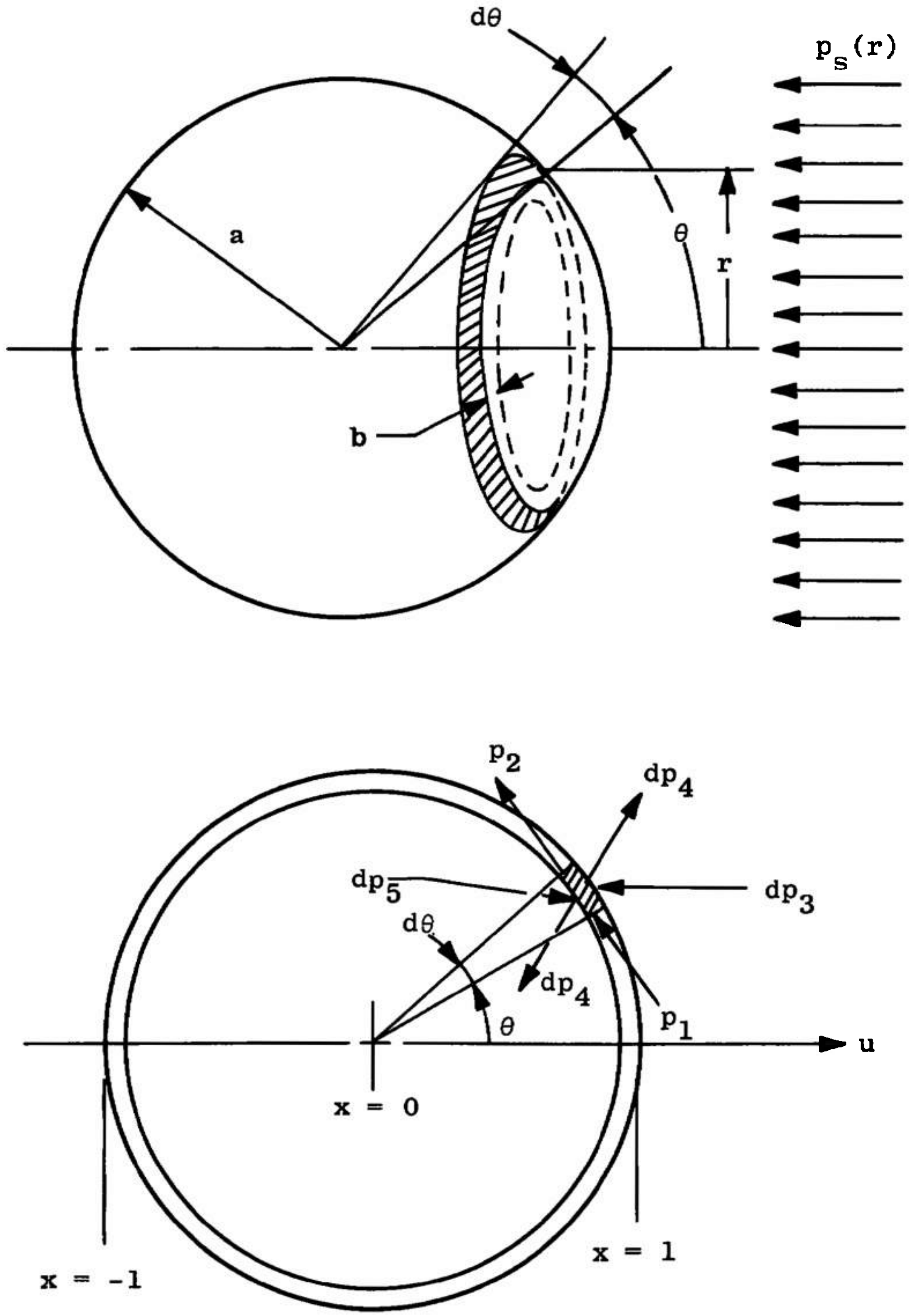

Fig. 1 Geometry and Nomenclature

The general equation governing the system can be obtained by performing a heat balance on the sector of the sphere between θ and $\theta + d\theta$ and the equation

$$dp_{in} = cD\, dV \frac{\partial T}{\partial t} \tag{1}$$

The rate of heat transfer into the sector at θ by conduction (Fig. 1) is

$$p_1 = -kA^k \frac{1}{a} \frac{\partial T}{\partial \theta}\bigg|_{\theta}$$

The rate of heat transfer out of the sector by conduction at $\theta + d\theta$ is

$$p_2 = -kA^k \frac{1}{a} \frac{\partial}{\partial \theta}\bigg|_{\theta+d\theta}$$

$$= -kA^k \frac{1}{a} \frac{\partial T}{\partial \theta}\bigg|_{\theta+d\theta} - \frac{k}{a}\frac{\partial}{\partial \theta}\left(A^k \frac{\partial T}{\partial \theta}\right) d\theta$$

Thus the total rate of heat transfer into the sector by conduction is

$$(p_1 - p_2) = \frac{k}{a}\left[\frac{dA^k}{d\theta}\frac{\partial T}{\partial \theta} + A^k \frac{\partial^2 T}{\partial \theta^2}\right] d\theta \tag{2}$$

Let q_S be the average over the projected area of the sphere of p_S; then

$$q_s = \frac{1}{\pi a^2}\int_0^a p_s(r)\, 2\pi\, r\, dr$$

or

$$q_s = \frac{2}{a^2}\int_0^a r\, p_s(r)\, dr \tag{3}$$

Now define the dimensionless functions

$$f(\theta) = \frac{1}{q_s} p_s (a \sin \theta) \tag{4}$$

and

$$h(\theta) = \begin{cases} 1 & 0 \le \theta < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le \theta \le \pi \end{cases} \tag{5}$$

The rate of heat transfer into the sector from the parallel radiation source is then

$$dp_3 = \alpha_s\, p_s(r)\, dA^p$$

or

$$dp_3 = \alpha_s q_s f(\theta)\, h(\theta)\, dA^p \tag{6}$$

The rate of heat transfer emitted out of the sector is given by

$$dp_4 = (\epsilon_i - \epsilon_o)\,\sigma\, T^4\, dA^R \tag{7}$$

The rate of heat transfer into the sector from radiation inside the sphere is given by

$$dp_5 = \alpha_i \int_0^{\pi} \epsilon_i\, \sigma\, T^4(\theta')\, dG_{\theta' - \theta}\, dA^R_{\theta'} \tag{8}$$

where the integration is of the area at θ'.

The different areas are given by

$$A^k = 2\pi\, a\, b \sin\theta \tag{9}$$

$$dA^p = \pi a^2\, d(\sin^2\theta)$$

or

$$dA^p = 2\pi a^2 \sin\theta \cos\theta\, d\theta \tag{10}$$

and

$$dA^R = 2\pi a^2 \sin\theta\, d\theta \tag{11}$$

The volume is

$$dV = b\, dA^R$$

or

$$dV = 2\pi a^2 b \sin\theta\, d\theta \tag{12}$$

It is proved in Appendix I that

$$dG_{\theta' - \theta} = \frac{1}{2\alpha_i} \sin\theta\, d\theta \tag{13}$$

A substitution in Eq. (1) of $dp_{in} = (p_1 - p_2) + dp_3 - dp_4 + dp_5$ results in

$$2\pi a\, b \frac{k}{a}\left[\cos\theta \frac{\partial T}{\partial \theta} + \sin\theta \frac{\partial^2 T}{\partial \theta^2}\right] d\theta$$

$$+\ 2\pi a^2 \alpha_s q_s f(\theta)\, h(\theta) \sin\theta \cos\theta\, d\theta$$

$$-\ 2\pi a^2 (\epsilon_i + \epsilon_o)\,\sigma T^4 \sin\theta\, d\theta$$

$$+\ \alpha_i\, \epsilon_i\, \sigma \left(\frac{1}{2\alpha_i} \sin\theta\, d\theta\right) 2\pi a^2 \int_0^{\pi} T^4(\theta') \sin\theta'\, d\theta'$$

$$=\ 2\pi a^2\, b\, c\, D \sin\theta\, d\theta\, \frac{\partial T}{\partial t}$$

Dividing by $\pi \sin\theta d\theta$ for $0<\theta<\pi$ this becomes

$$2bk\left[\cot\theta \frac{\partial T}{\partial \theta} - \frac{\partial^2 T}{\partial \theta^2}\right] + 2a^2 \alpha_s q_s f(\theta) h(\theta) \cos\theta$$
$$- 2a^2 (\epsilon_i + \epsilon_o) \sigma T^4 + a^2 \epsilon_i \sigma \int_0^\pi T^4(\theta') \sin\theta' d\theta' \tag{14}$$
$$= 2a^2 b c D \frac{\partial T}{\partial t}$$

Define the dimensionless variables

$$Z = \frac{T}{T_m} \tag{15}$$

and

$$y = \frac{t}{t_m} \tag{16}$$

where, for the present, T_m and t_m are, respectively, some arbitrary temperature and some arbitrary time. Equation (14) is obtained in dimensionless form by using Eqs. (15) and (16) and dividing through by $2bkT_m$. The result is

$$\cot\theta \frac{\partial Z}{\partial \theta} + \frac{\partial^2 Z}{\partial \theta^2} + \left(\frac{a^2 \alpha_s q_s}{b k T_m}\right) f(\theta) h(\theta) \cos\theta$$
$$- \left(\frac{a^2 \epsilon_o \sigma T^3{}_m}{bk}\right) \left(1 + \frac{\epsilon_i}{\epsilon_o}\right) Z^4 + \frac{1}{2}\left(\frac{a^2 \epsilon_o \sigma T^3{}_m}{b k}\right) \left(\frac{\epsilon_i}{\epsilon_o}\right) \int_0^\pi Z^4(\theta') \sin\theta' d\theta' \tag{17}$$
$$= \left(\frac{a^2 c D}{k t_m}\right) \frac{\partial Z}{\partial y}$$

This introduces three dimensionless quantities which will be defined

$$N_s = \frac{a^2 \alpha_s q_s}{b k T_m} \tag{18}$$

$$N_R = \frac{a^2 \epsilon_o \sigma T_m^3}{b k} \tag{19}$$

and

$$N_c = \frac{a^2 c D}{k t_m} \tag{20}$$

These quantities may be used as a measure of the effects of an external energy source, external radiation, and the heat capacity, respectively, as compared with the heat transfer by conduction. Also define

$$E = \frac{\epsilon_i}{\epsilon_o} \tag{21}$$

In these terms, Eq. (17) becomes

$$\frac{\partial^2 Z}{\partial \theta^2} + \cot\theta \frac{\partial Z}{\partial \theta} - N_s f(\theta) h(\theta) \cos\theta$$

$$+ N_R \left[\frac{1}{2} E \int_0^\pi Z^4(\theta') \sin\theta' d\theta' - (1 + E) Z^4 \right] \tag{22}$$

$$= N_c \frac{\partial Z}{\partial y}$$

This equation can be written in a different form by a change of variable

$$x = \cos\theta$$

From the equation

$$\frac{\partial Z}{\partial \theta} = \frac{\partial Z}{\partial x} \frac{dx}{d\theta} = -\sin\theta \frac{\partial Z}{\partial x}$$

is obtained

$$\cot\theta \frac{\partial Z}{\partial \theta} = -\cos\theta \frac{\partial Z}{\partial x} = -x \frac{\partial Z}{\partial x}$$

Also

$$\frac{\partial^2 Z}{\partial x^2} = \frac{\partial}{\partial \theta}\left(-\sin\theta \frac{\partial Z}{\partial x}\right)$$

$$= -\cos\theta \frac{\partial Z}{\partial x} + \sin^2\theta \frac{\partial^2 Z}{\partial x^2}$$

$$= -x \frac{\partial Z}{\partial x} + (1 - x^2) \frac{\partial^2 Z}{\partial x^2}$$

Defining

$$F(x) = f(\cos^{-1} x) \tag{24}$$

and

$$H(x) = \begin{cases} 1 & 0 < x \leq 1 \\ 0 & -1 \leq x \leq 0 \end{cases} \tag{25}$$

Thus Eq. (22) becomes

$$(1 - x^2) \frac{\partial^2 Z}{\partial x^2} - 2x \frac{\partial Z}{\partial x} + N_s F(x) H(x) x$$

$$+ N_R \left[\frac{1}{2} E \int_{-1}^1 Z^4(x') dx' - (1 + E) Z^4 \right] \tag{26}$$

$$= N_c \frac{\partial Z}{\partial y}$$

Either Eq. (22) or Eq. (26) describes the temperature of the system as a function of position and time. Given an initial temperature distribution, the numerical solution of these equations could be obtained.

2.2 STEADY STATE EQUATIONS

If steady state condition is assumed, a simplification of Eqs. (22) and (26) can be made. Performing a heat balance on the whole sphere at steady state gives

$$\int_0^{\pi} \epsilon_0 \, \sigma \, T^4 \, dA^R = \pi a^2 \, \alpha_s \, q_s \tag{27}$$

or

$$2\pi a^2 \epsilon_0 \, \sigma \int_0^{\pi} T^4 \sin\theta \, d\theta = \pi a^2 \, \alpha_s \, q_s$$

from which is obtained

$$\int_0^{\pi} T^4 \sin\theta \, d\theta = \frac{\alpha_s \, q_s}{2\epsilon_0 \, \sigma} \tag{28}$$

or

$$\int_0^{\pi} Z^4 \sin\theta \, d\theta = \frac{\alpha_s \, q_s}{2\epsilon_0 \, \sigma \, T_m^4} \tag{29}$$

If T_m is defined as the temperature such that if the sphere was isothermal, it would radiate the same power; then

$$4\pi a^2 \, \epsilon_0 \, \sigma \, T_m^2 = \int_0^{\pi} \epsilon_0 \, \sigma \, T^4 \, dA^R$$

$$= \pi a^2 \, \alpha_s \, q_s$$

or

$$T_m = \sqrt[4]{\frac{\alpha_s \, q_s}{4 \, \epsilon_0 \, \sigma}} \tag{30}$$

With this definition, Eq. (29) becomes

$$\int_0^{\pi} Z^4 \sin\theta \, d\theta = 2 \tag{31}$$

Thus

$$\int_{-1}^{1} Z^4 \, dx = 2 \tag{32}$$

Also, for steady state conditions a relation between N_S and N_R can be obtained. By Eq. (18)

$$N_s = \frac{a^2 \alpha_s \, q_s \, T_m^3}{b \;\, k \; T_m^4}$$

$$= \frac{a^2 \alpha_s \, q_s \, T_m^3}{b \, K \left(\dfrac{\alpha_s \, q_s}{4\epsilon_0 \, \sigma}\right)}$$

$$= \frac{4 a^2 \epsilon_0 \;\, \sigma \, T_m^3}{bk}$$

or

$$N_s = 4N_R \tag{33}$$

Using these substitutions and noting that for steady state the right hand side is zero and the partial derivatives become total derivatives, Eq. (22) becomes

$$\frac{\partial^2 Z}{\partial \theta^2} + \cot\theta \frac{\partial Z}{\partial \theta} + 4N_R\, f(\theta)\, h(\theta) \cos\theta$$

$$+ N_R \left(\frac{1}{2}\right) E\,(2) - N_R (1 + E) Z^4 = 0$$

or

$$\frac{\partial^2 Z}{\partial \theta^2} + \cot\theta \frac{\partial Z}{\partial \theta} + N_R \left\{ 4f(\theta)\, h(\theta) \cos\theta + E - (1 + E) Z^4 \right\} = 0 \tag{34}$$

Similarly Eq. (26) becomes

$$(1 - x^2) \frac{\partial^2 Z}{\partial x^2} - 2x \frac{\partial Z}{\partial x} + N_R \left\{ 4F(x)\, H(x)\, x + E - (1 + E) Z^4 \right\} = 0 \tag{35}$$

Along with the simplification in the equations is a complication in the condition to be satisfied by the solution. Whereas the condition to be satisfied by the solution of the transient equations was simply the initial temperature distribution, the solution of Eq. (34) must satisfy Eq. (31) and the solution for Eq. (35) must satisfy Eq. (32). This complication in side conditions is not peculiar to this particular system but is a general circumstance when going from the transient equations to the steady state equations of most systems.

SECTION III
SOLUTION OF THE STEADY STATE EQUATIONS

3.1 NUMERICAL METHOD

This section describes the numerical method used to obtain numerical solutions to the steady state equations. A steady state solution could be obtained by assuming an initial temperature distribution and obtaining a numerical solution of the transient equations. However, a more efficient method is to assume a starting value of the steady state equation, obtain a solution, and iterate to find the solution which fits the side condition. This method is described below as applied to Eq. (35).

Define

$$S = \frac{\partial Z}{\partial x} \tag{36}$$

and Eq. (35) becomes

$$(1 - x^2)\frac{dS}{dx} - 2xS + N_R\left\{4F(x)\,H(x)\,x + E - (1 + E)\,Z^4\right\} = 0$$

Also define

$$R(x) = \int_{-1}^{x} Z^4\,dx \tag{37}$$

From these equations, we easily obtain the system of equations

$$\frac{dR}{dx} = Z^4 \tag{38}$$

$$\frac{dS}{dx} = \frac{1}{1 - x^2}\left\{2xS + N_R\left[(1 + E)\,Z^4 - 4F(x)\,H(x)\,x - E\right]\right\} \tag{39}$$

$$\frac{dZ}{dx} = S \tag{40}$$

A numerical solution of this system of equations could easily be obtained if the values of R, S, and Z at x = -1 were known. However, instead the following side conditions must be met. From Eq. (37)

$$R(-1) = 0 \tag{41}$$

From Eq. (32) is obtained

$$R(1) = 2 \tag{42}$$

and from Eq. (35) is obtained the condition

$$S(-1) = \frac{1}{2}N_R\left\{(1 - E)\left[Z(-1)\right]^4 - E\right\} \tag{43}$$

From the physical orientation, the coldest spot on the sphere is at x = -1; thus, S(-1) is positive or zero and from Eq. (43) is obtained

$$Z(-1) \geq \left(\frac{E}{1 + E}\right)^{1/4}$$

Also with this assumption, from Eq. (32),

$$Z(-1) \leq 1$$

If Z_1 and Z_u are lower and upper bounds of Z(-1) then by the above inequalities valid values are

$$Z_\ell = \left(\frac{E}{1 + E}\right)^{1/4}$$

and

$$Z_u = 1$$

At the beginning of each iteration a value, Z_o, is defined as

$$Z_o = \frac{1}{2}\left(Z_\ell + Z_u\right)$$

and starting values

$$R(-1) = 0$$

$$S(-1) = \frac{1}{2} N_R \left[(1 - E) Z_o^4 - E\right]$$

$$Z(-1) = Z_o$$

are used. It is seen that the side conditions, Eqs. (41) and (43), are satisfied. Iteration is continued until Eq. (42) is satisfied within a given tolerance, ΔR. That is until

$$2 - \Delta R \leq R(1) \leq 2 + \Delta R$$

Each iteration determines whether Z_o is an upper or lower bound for $Z(-1)$. It is assumed that if Z_o is too low, then the solution obtained for $Z(x)$ is below the correct solution and that if Z_o is too high, then the solution obtained is above the correct solution. (This assumption was verified by numerical results.) If at any point in a solution

$$Z(x) < Z_\ell$$

then it is known that Z_o is a lower bound for $Z(-1)$. Therefore Z_ℓ is set equal to Z_o and another iteration is begun. If at any point in a solution

$$R(x) > 2 + \Delta R$$

then it is known that Z_o is an upper bound, so Z_u is set equal to Z_o and a new iteration is begun. If a solution proceeds to $X = 1$, then $R(1)$ is tested to see if

$$R(1) < 2 - \Delta R$$

If so, then Z_ℓ is set equal to Z_o and another iteration begun. If not, then Eq. (42) is satisfied within the given tolerance and the desired solution has been obtained.

A flow chart of this method is shown in Fig. 2.

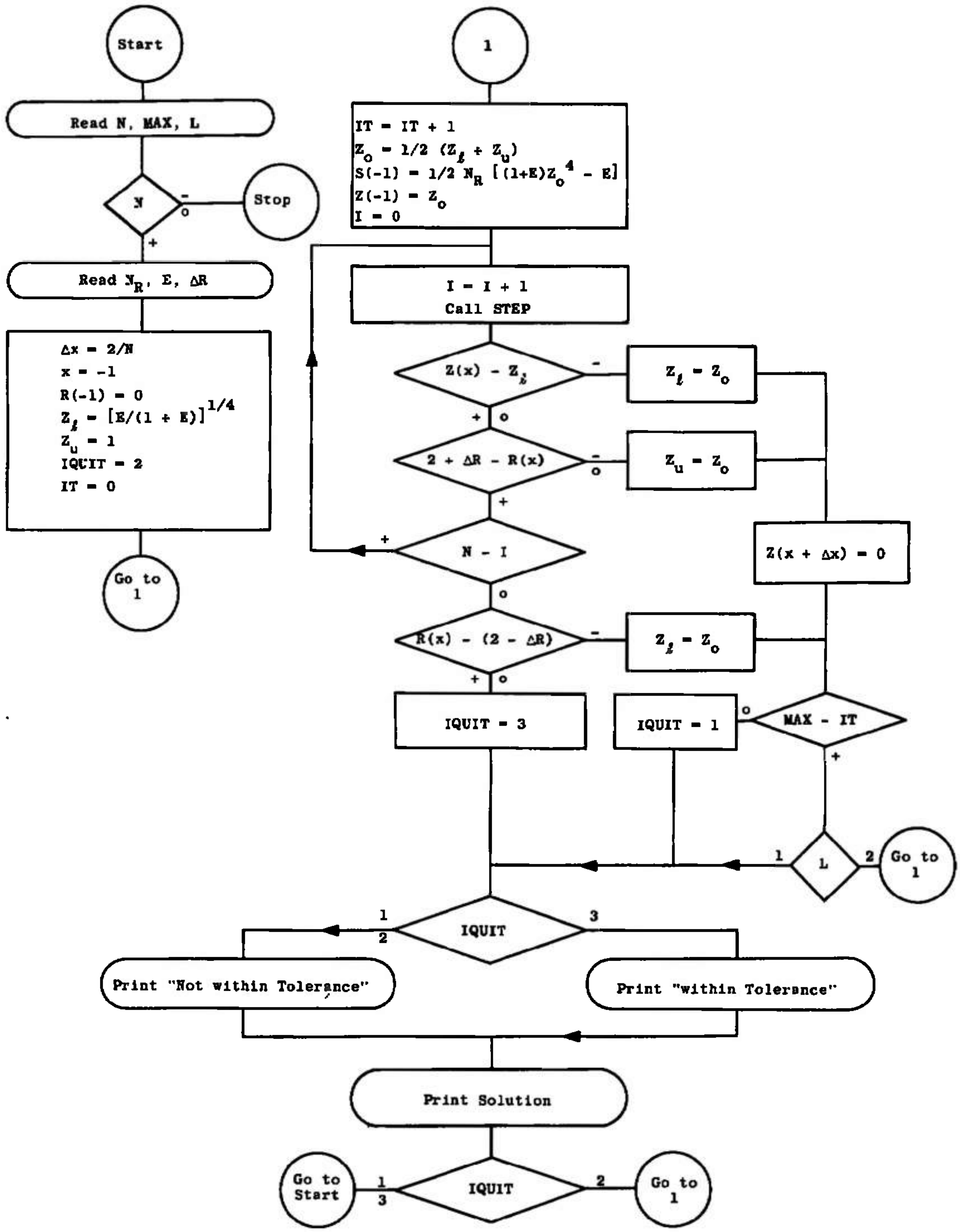

Fig. 2 Flow Chart of Main Program

3.2 COMPUTER PROGRAM

A computer program was written in FORTRAN II for the SDS 920 computer to solve the steady state equation employing the method described above. The main program follows the flow chart shown in Fig. 2. The subroutine STEP called by the main program is a standard subroutine used to solve differential equations. STEP calls another subroutine FUN which evaluates Eqs. (38), (39), and (40). These are explained in detail below.

3.2.1 Main Program

The input to the program is N, MAX, and L defined below and N_R, E, and ΔR. N is the number of steps. Since the solution is required for $-1 \leq x' \leq 1$, the step size is

$$\Delta x = 2/N$$

MAX contains a limit to the number of iterations. The fractional portion of the SDS 920 floating point number is 39 binary digits. Since the numerical method halves the interval containing Z(-1) each iteration, the computer precision limits the method to 39 iterations. If L is 1, then the solution is printed after every iteration. If L is 2, then only the last solution is printed.

The flow chart for the main program is shown in Fig. 2, and the listing is given in Appendix II.

3.2.2 Subroutine STEP

Subroutine STEP is a program of the Runge-Kutta one step method of solving differential equations. This method is explained in Ref. 4.

Given a system of differential equations

$$\frac{dy^i}{dx} = f^i(x, y^1, y^2, \ldots, y^n), \; i = 1, 2, \ldots, n \tag{44}$$

and starting values

$$y^i(x_o) = y_o^i$$

A one step method is an algorithm which uses the differential equations and the starting values to find an approximation to the solution at $x_o + \Delta x$

$$y^i(x_o + \Delta x) \approx y_1^i$$

These values can be used as new starting values to find an approximation to the solution at $x_o + 2\Delta x$ or in general, after p steps

$$y^i(x_p) \approx y^i_p$$

where

$$x_p = x_o + p\,\Delta x$$

The smaller the step-size, Δx, is the better the approximation becomes. The accuracy is limited only by the round off error which depends on the precision of the computer.

Subroutine STEP is called by the statement

CALL STEP (N, X, Y, DX)

where Y is dimensioned Y(25). N contains the number of equations and DX the step size. Suppose p steps have been taken. Then, when STEP is called, X contains x_p and Y contains y^i_p, with i = 1, 2, ... n. On return, X contains x_{p+1} and Y contains y^i_{p+1}, with i = 2, ... n. STEP calls on a subroutine FUN to evaluate Eq. (44).

The listing of STEP is given in Appendix II.

3.2.3 Subroutine FUN

In the present case, subroutine FUN was written to evaluate Eqs. (38), (39), and (40) which correspond to Eq. (44) for this problem. It is seen that Eq. (39) is singular at $x = \pm 1$. To avoid this difficulty if $x < -1 + 0.1\,\Delta x$, then it is assumed that

$$1 - x^2 \approx 1 - x_1^2$$

where $x_1 = -1 + 0.1\,\Delta x$. If $x > 1 - 0.1\Delta x$, then the above approximation is again used only this time $x_1 = 1 - 0.1\Delta x$.

The flow chart of FUN is shown in Fig. 3, and the listing is given in Appendix II.

SECTION IV
RESULTS AND DISCUSSION

The computer program described above was used to obtain solutions of Eq. (35) for various values of N_R and E. The convergence

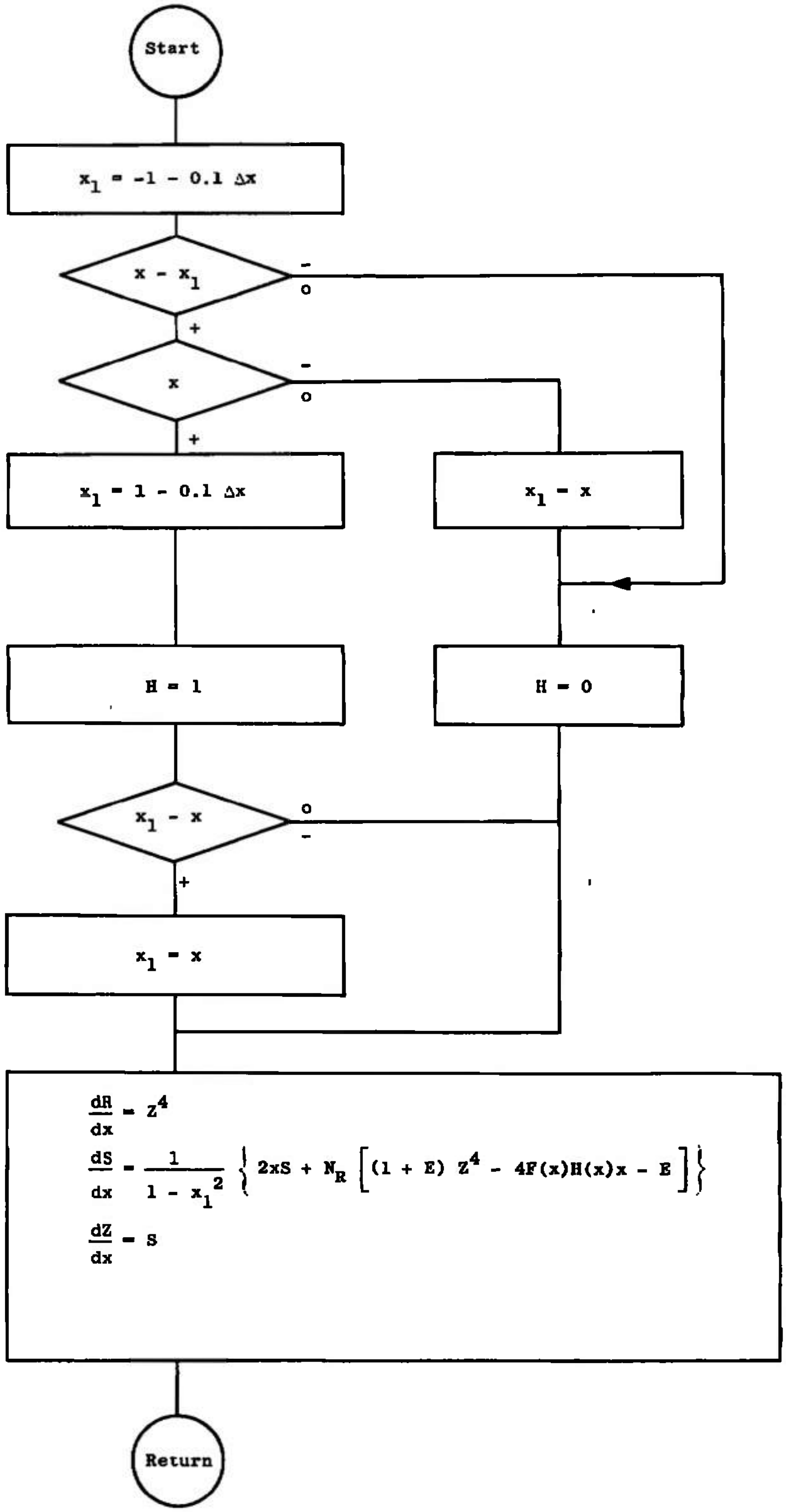

Fig. 3 Flow Chart of FUN

of the iterations is illustrated in Fig. 4. Figures 5, 6, and 7 are the graphs of solutions with $N_R = 1$, 2, and 3, respectively, for various values of E. The solution with N_R infinite can be obtained from Eq. (35), where for this case, Z^4 is a linear function of x. This linearity is shown in Fig. 8, and the Z versus x relationship is shown in Fig. 9. The case, $N_R = 0$, can arise only from infinite conductivity since it is assumed that $b \ll a$. This means the sphere would be isothermal giving the solution, $Z(x) = 1$, independent of E. It is interesting to note that at $x = 0.25$, the dimensionless temperature Z is very close to one in all cases. The solutions show that when $E \geq 1$, the temperature distribution changes relatively slow as N_R is changed. That is, the temperature distribution is determined largely by internal radiation. For $E < 1$, N_R has a greater influence on the solutions, implying that conduction also is an important factor in determining the temperature distribution.

If N_R and E are the same for two different systems, then the same equation describes both systems; thus, the temperature distribution of one system can be inferred from the measured temperature distribution of the other system. This is the basis for thermal modeling. The observations made above, on the behavior of the solutions of Eq. (35), imply that for $E \geq 1$ the tolerance of duplication of N_R in thermal modeling can be relaxed, but when $E < 1$ the requirement for duplication of N_R becomes more stringent.

In thermal modeling it may be desired to preserve temperature, since the thermal properties may be a function of temperature. For this case, in addition to preserving N_R and E, one must preserve T_m which is given by Eq. (30). Thus, from Eqs. (19), (21), and (30), after cancelling constants and T_m, are obtained the requirements

$$\left(\frac{a^2 \epsilon_o}{b\,k}\right)_m = \left(\frac{a^2 \epsilon_o}{b\,k}\right)_p \tag{45}$$

$$\left(\frac{\epsilon_i}{\epsilon_o}\right)_m = \left(\frac{\epsilon_i}{\epsilon_o}\right)_p \tag{46}$$

$$\left(\frac{\alpha_s q_s}{\epsilon_o}\right)_m = \left(\frac{\alpha_s q_s}{\epsilon_o}\right)_p \tag{47}$$

where the subscripts m and P refer to model and prototype. If furthermore, materials and surface properties are preserved, then Eqs. (46) and (47) are automatically fulfilled and from Eq. (45) the well-known scaling requirement

$$\left(\frac{a^2}{b}\right)_m = \left(\frac{a^2}{b}\right)_p \tag{48}$$

is obtained.

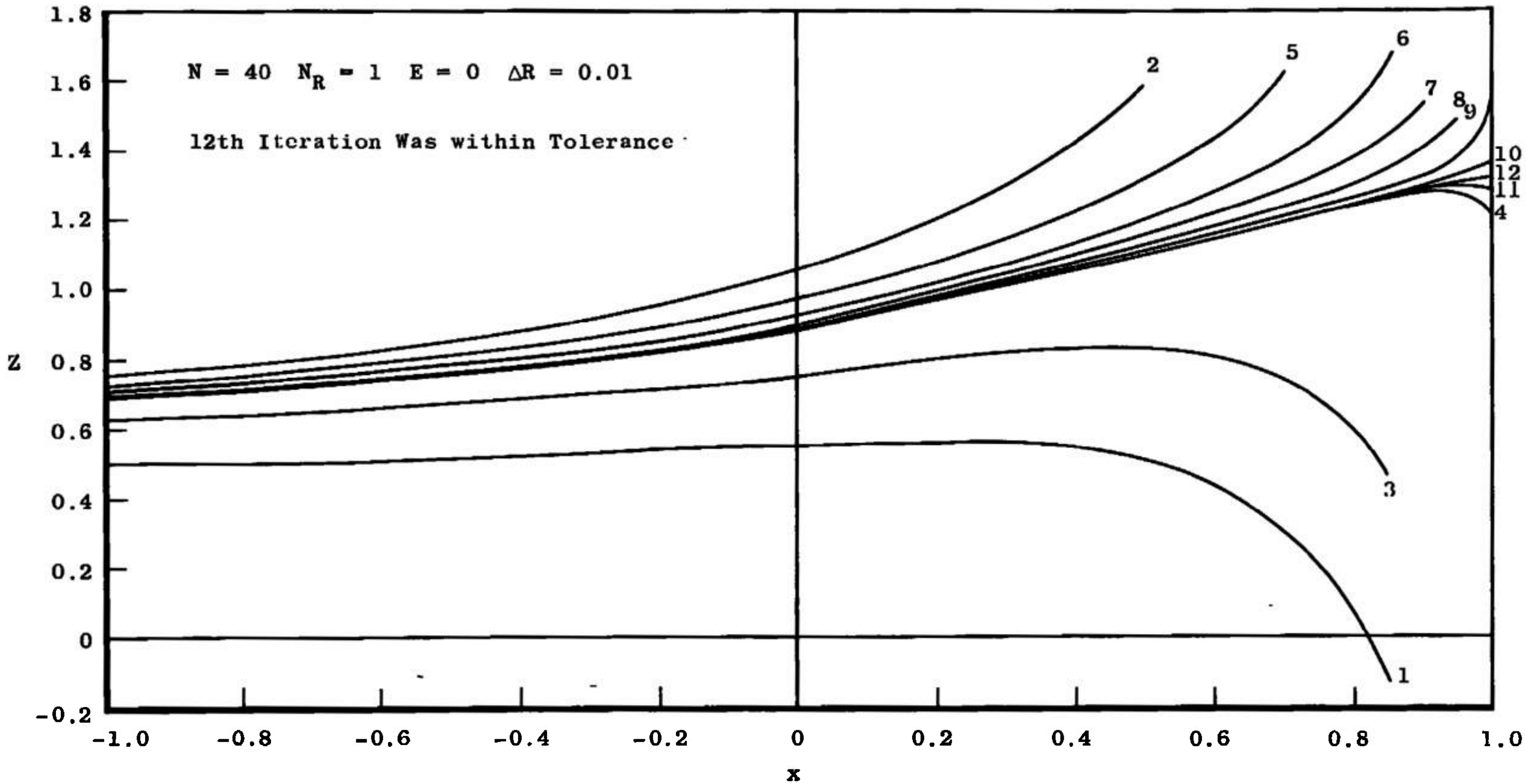

Fig. 4 Convergence of the Iterations

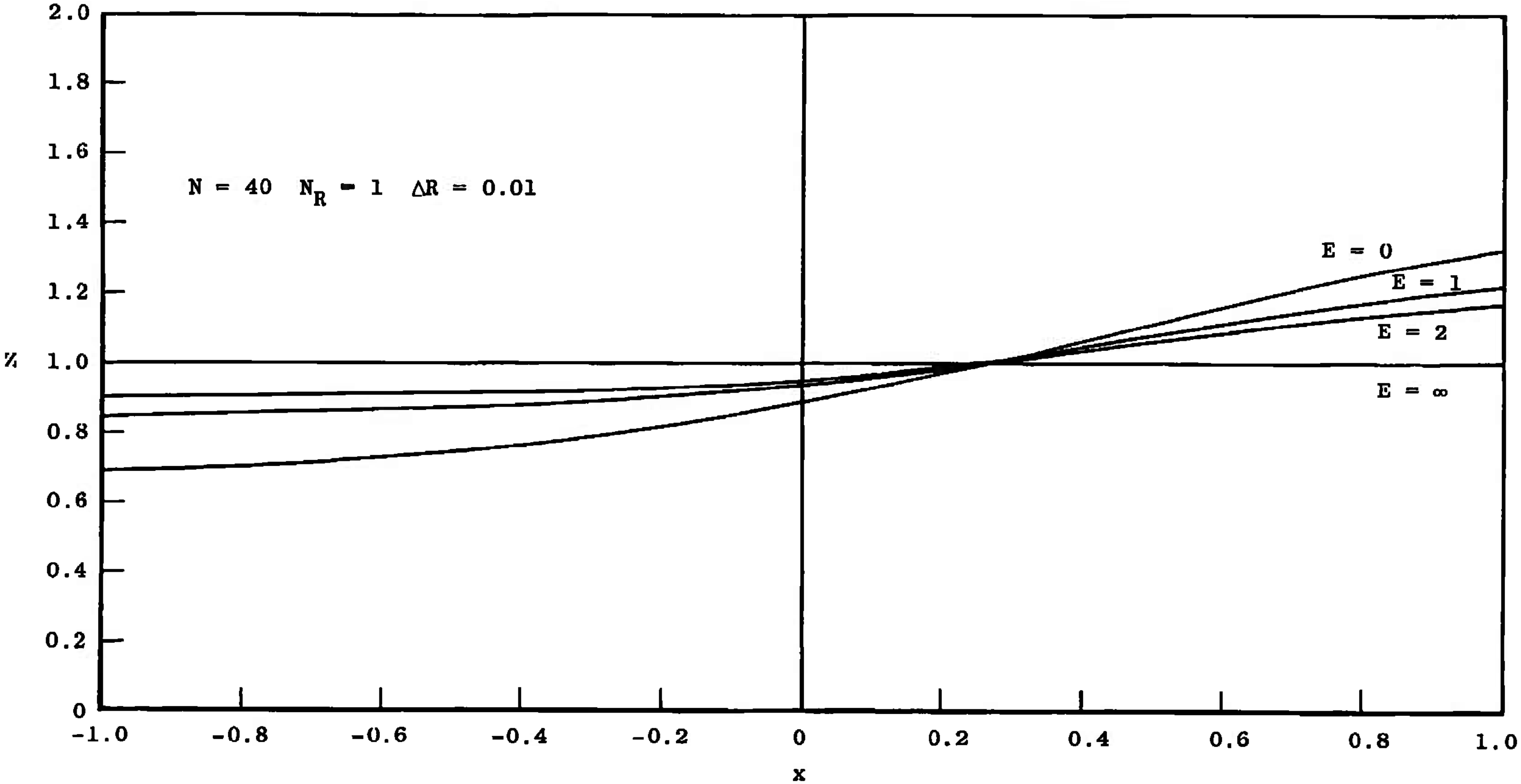

Fig. 5 Solutions with $N_R = 1$

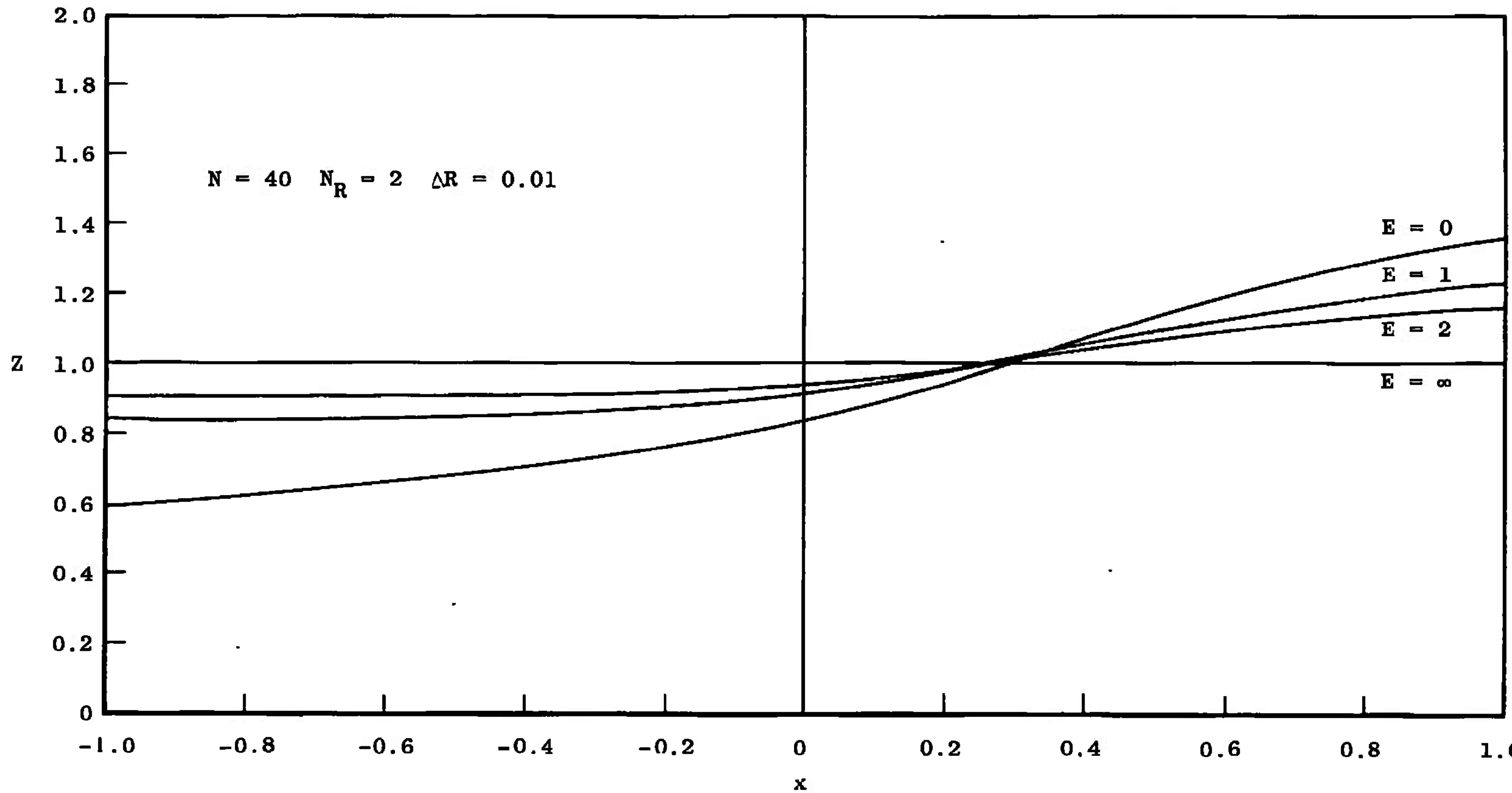

Fig. 6. Solutions with N_R = 2

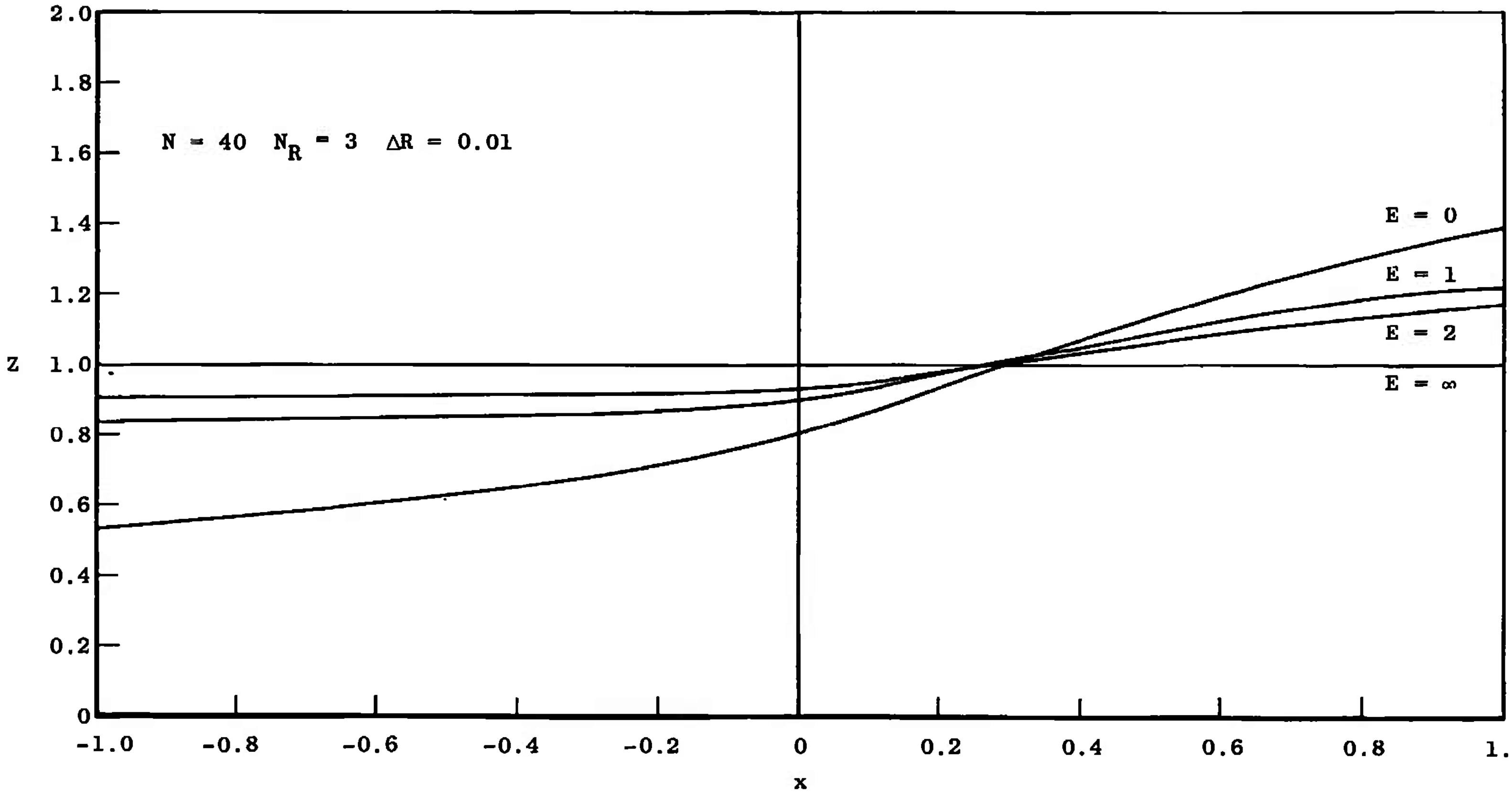

Fig. 7 Solutions with N_R = 3

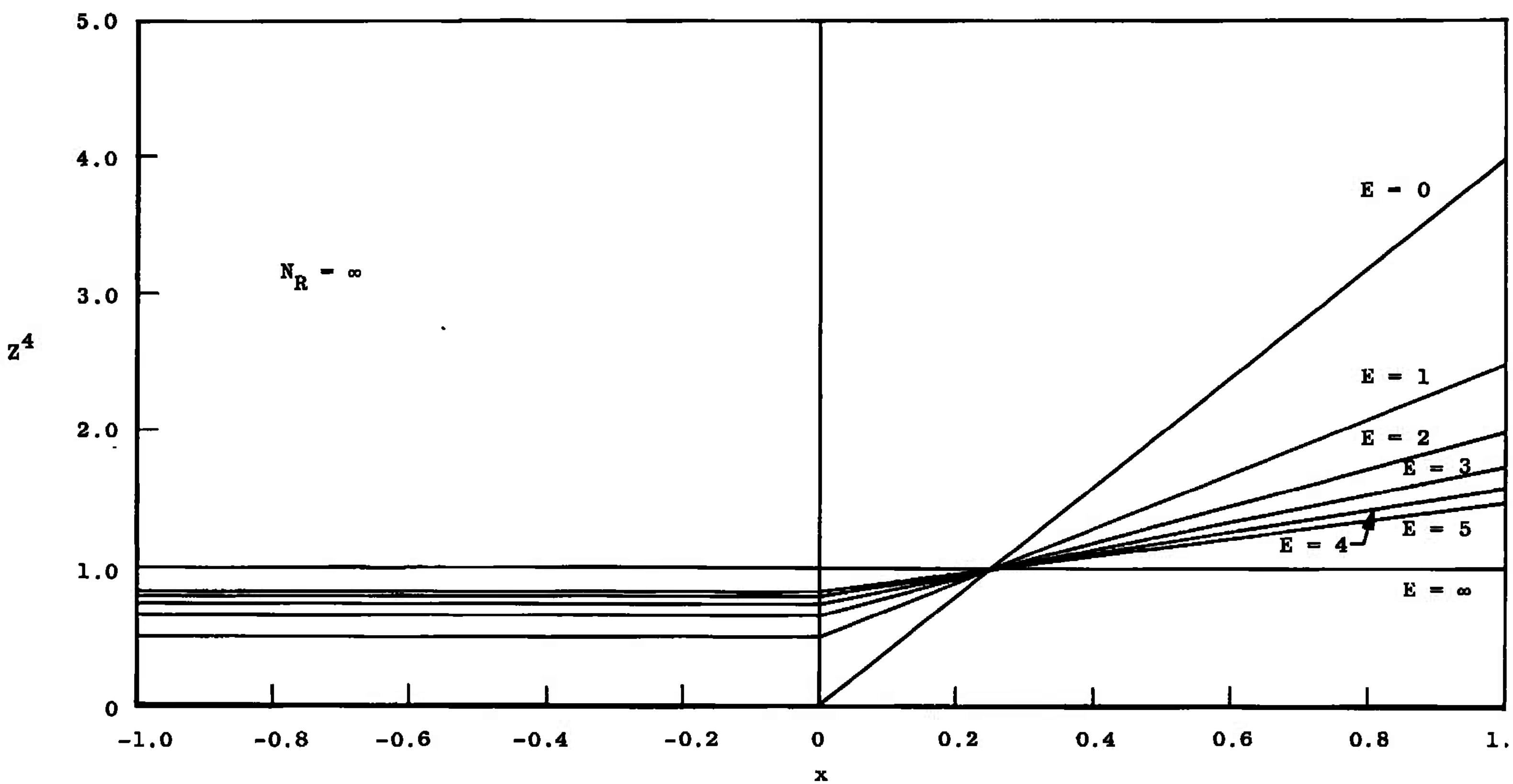

Fig. 8 Linearity of Z^4 when N_R is Infinite

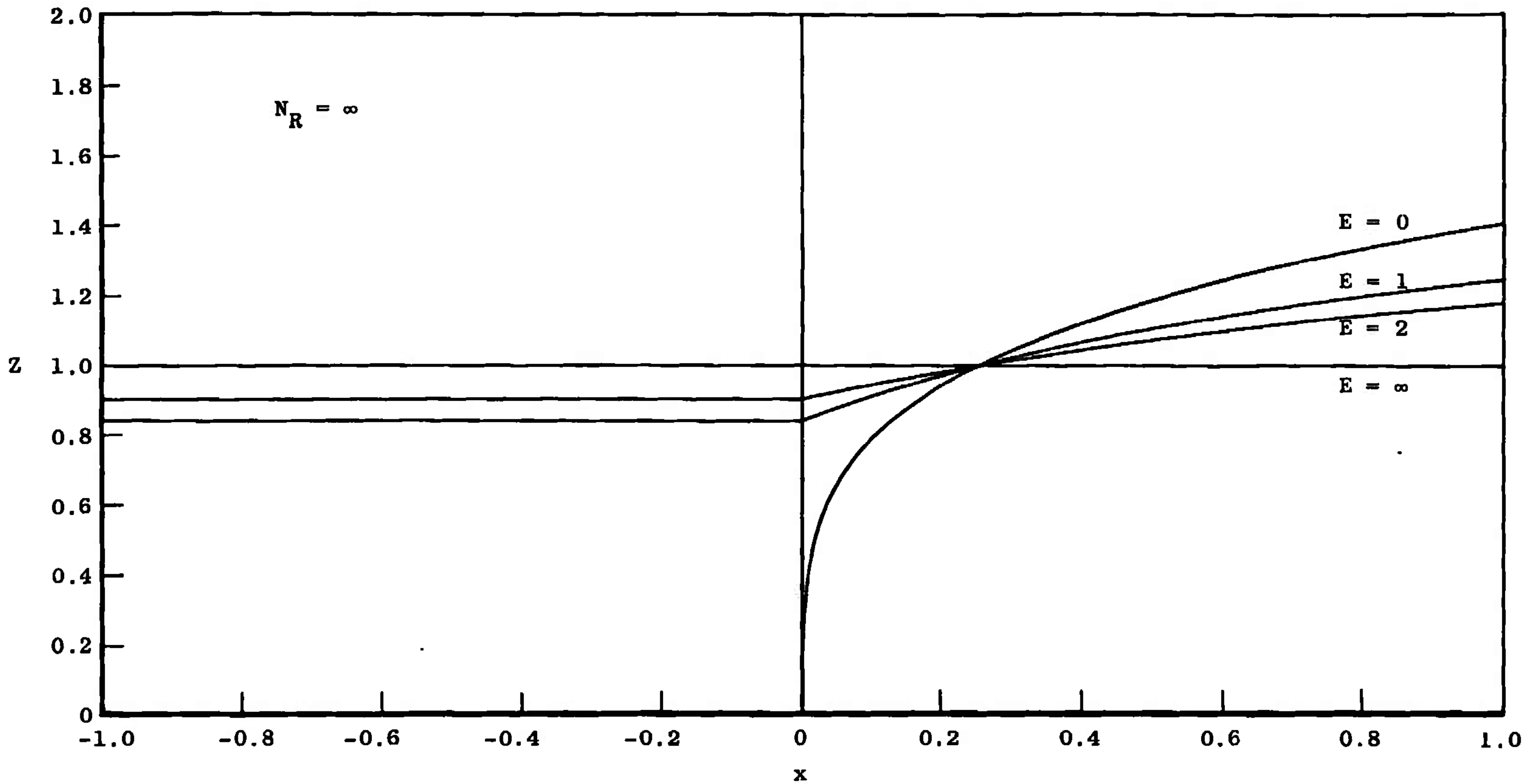

Fig. 9 Solutions with N_R Infinite

There is, at least theoretically, a possibility of modeling without temperature preservation. The preservation of N_R requires that

$$\frac{\left(T_m^3\right)_m}{\left(T_m^3\right)_P} = \frac{\left(\frac{b\,k}{a^2 \epsilon_o}\right)_m}{\left(\frac{b\,k}{a^2 \epsilon_o}\right)_P} \tag{49}$$

But by Eq. (30)

$$\frac{\left(T_m^4\right)_m}{\left(T_m^4\right)_P} = \frac{\left(\frac{\alpha_s q_s}{\epsilon_o}\right)_m}{\left(\frac{\alpha_s q_s}{\epsilon_o}\right)_P} \tag{50}$$

Thus, from Eqs. (49) and (50) is obtained the requirement

$$\left[\frac{\left(\frac{b\,k}{a^2}\right)_m}{\left(\frac{b\,k}{a^2}\right)_P}\right]^{1/3} = \left[\frac{\left(\alpha_s\, q_s\, \epsilon_o^{\,1/3}\right)_m}{\left(\alpha_s\, q_s\, \epsilon_o^{\,1/3}\right)_P}\right]^{1/4} \tag{51}$$

The preservation of E requires that

$$\frac{(\epsilon_i)_m}{(\epsilon_i)_P} = \frac{(\epsilon_o)_m}{(\epsilon_o)_P} \tag{52}$$

Given a prototype, after a suitable choice of dimensions and selection of material for the model, the left-hand side of Eq. (51) is determined. The choice of surface finish on the model will determine $(\alpha_s)_m$ and $(\epsilon_o)_m$. It is then possible to adjust $(q_s)_m$ to fulfill Eq. (51). Then $(\epsilon_i)_m$ is determined by Eq. (52). Once the temperature distribution of the model is measured, the temperature of the prototype can be calculated by Eq. (15).

$$\frac{(T)_P}{(T_m)_P} = \frac{(T)_m}{(T_m)_m} \tag{53}$$

since Z is preserved by design. Equation (30) is used to calculate T_m.

SECTION V
SUMMARY

The equation governing the heat transfer of a spherical shell subjected to parallel radiation was derived, conduction and radiation being considered. A numerical method was developed to solve the steady state equation, and a computer program was described which employed the method. Solutions of the steady state equation were graphically presented.

The requirements for temperature preservation in thermal modeling were derived. The possibility of thermal modeling without temperature preservation was discussed. It was observed that for an inside emissivity to outside emissivity ratio greater than one, the requirement for duplication of the other dimensionless ratio can be relaxed.

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APPENDIXES

I. DERIVATION OF FORM-SURFACE FACTOR

II. FORTRAN LISTING OF COMPUTER PROGRAM

APPENDIX I
DERIVATION OF FORM-SURFACE FACTOR

The fact that flux leaving any part of the interior of a sphere diffusely spreads uniformly over the interior of the sphere implies that the form factor from the area at θ to the area at θ'

$$F_{\theta-\theta'} = \frac{dA^R_{\theta'}}{4\pi a^2}$$

Also, the form factor from the sphere to the area at θ' is

$$F_{S-\theta'} = \frac{dA^R_{\theta'}}{4\pi a^2}$$

Consider flux leaving the area at θ. $F_{\theta-\theta'}$ of this flux reaches θ' directly. The flux from any reflection leaves the sphere uniformly so $F_{s-\theta'}$ of the reflected flux reaches θ'.

Of the flux leaving θ, the flux eventually striking θ' is, by adding

$$\begin{aligned} G_{\theta-\theta'} &= F_{\theta-\theta'} + \rho_i F_{s-\theta'} + \rho_i^2 F_{s-\theta'} + \cdots \\ &= \frac{dA^R_{\theta'}}{4\pi a^2}\left(1 + \rho_i + \rho_i^2 + \cdots\right) \\ &= \frac{dA^R_{\theta'}}{4\pi a^2 (1 - \rho_i)} \end{aligned}$$

or from Eq. (11) this becomes

$$G_{\theta-\theta'} = \frac{\sin\theta' d\theta'}{2a_i}$$

This is the fraction of the flux leaving the area at θ which eventually strikes the area at θ' and is called the form-surface factor.

APPENDIX II
FORTRAN LISTING OF COMPUTER PROGRAM

```
C 5027  RADIATING SPHERE  D.C. TODD  1-12-67
      DIMENSION X(100),Z(100),ZF(100),R(100),S(100),DS(100),V(3),DUM
      COMMON D,E,DX,ZFO,DSO
C
C INPUT AND INITIATE
C
    1 ACCEPT TAPE 1001,N,MAX,L
      IF(N)2,2,3
    2 PRINT 1004
      STOP
    3 ACCEPT TAPE 1002,D,E,T
      DX=2./N
      X(1)=-1.
      R(1)=0.
      ZU=1.
      ZL=(E/(1.+E))**.25
      IQUIT=2
      IT=0
C
C START OF ITERATION
C
    4 IT=IT+1
      ZO=.5*(ZU+ZL)
      S(1)=.5*D*((1.+E)*ZO**4-E)
      Z(1)=ZO
      U=-1.
      V(1)=0.
      V(2)=S(1)
      V(3)=ZO
      CALL FUN(U,V,DUM)
      ZF(1)=ZFO
      DS(1)=DSO
C FIND SOLUTION
      DO 5  I=1,N
      CALL STEP(3,U,V,DX)
      X(I+1)=U
      Z(I+1)=V(3)
      ZF(I+1)=ZFO
      R(I+1)=V(1)
      S(I+1)=V(2)
      DS(I+1)=DSO
      IF(V(3)-ZL)50,50,60
   60 C=2.-V(1)
      IF(T+C)51,51,5
    5 CONTINUE
C
C TEST TO SEE IF THIS IS LAST ITERATION
C
      IF(T-C)50,6,6
    6 IQUIT=3
      GO TO 10
   50 ZL=ZO
      GO TO 52
   51 ZU=ZO
```

```
   52 Z(I+2)=0
    7 IF(MAX-IT)8,8,9
    8 IQUIT=1
      GO TO 10
    9 GO TO(10,4),L
C
C START OUTPUT
C
   10 PRINT 1000
      PRINT 1005
      PRINT 1003,D,E,DX,T,ZL,ZU
      PRINT 1006,IT
      GO TO(11,11,12),IQUIT
   11 PRINT 1008
      GO TO 13
   12 PRINT 1007
   13 PRINT 1009
      PRINT 1003,(X(I),I=1,N+1)
      PRINT 1010
      PRINT 1003,(Z(I),I=1,N+1)
      PRINT 1011
      PRINT 1003,(ZF(I),I=1,N+1)
      PRINT 1012
      PRINT 1003,(R(I),I=1,N+1)
      PRINT 1013
      PRINT 1003,(S(I),I=1,N+1)
      PRINT 1014
      PRINT 1003,(DS(I),I=1,N+1)
      GO TO(1,4,1),IQUIT
 1000 FORMAT($1D:C. TODD  5027$)
 1001 FORMAT(40I2)
 1002 FORMAT(6E12.0)
 1003 FORMAT(1P10E12.4)
 1004 FORMAT(1H1)
 1005 FORMAT(1H04X2HNR10X1HE11X2HDX10X1HT11X2HZL10X2HZU/)
 1006 FORMAT($0ITERATIONS I3)
 1007 FORMAT($0WITHIN TOLERANCE$)
 1008 FORMAT($0NOT WITHIN TOLERANCE$)
 1009 FORMAT($0X$/)
 1010 FORMAT($0Z$/)
 1011 FORMAT($0FOURTH POWERS$/)
 1012 FORMAT($0INTEGRAL OF FOURTH POWERS$/)
 1013 FORMAT($0FIRST DERIVATIVES$/)
 1014 FORMAT($0SECOND DERIVATIVES$/)
COMMON ALLOCATION

  77776 D         77774 E         77772 DX        77770 ZFO
  77766 DSO

PROGRAM ALLOCATION

  00004 X         00314 Z         00624 ZF        01134 R
  01444 S         01754 DS        02264 V         02272 DUM
  02300 N         02301 MAX       02302 L         02303 IQUIT
  02304 IT        02305 I         02306 T         02310 ZU

  02312 ZL        02314 ZO        02316 U         02320 C
SUBPROGRAMS REQUIRED

  FUN       STEP
THE END
```

```
C ONE STEP BY RUNGE-KUTTA  D.C. TODD  1-12-67
      SUBROUTINE STEP]N,X,Y,DX]
      DIMENSION Y]25[,D[25[,YO(25],F[25[
      CALL FUN[X,Y,D]
      DO 1 ]=1,N
      YO[]]=Y[I[
      F[]]=D]]]
    1 Y[]]=YO[]]+.5*DX*D[I]
      X=X+.5*DX
      CALL FUN[X,Y,D]
      DO 2 ]=1,N
      F[]]=F[I]+2.*D[]]
    2 Y[]]=YO[]]+.5*DX*D[]]
      CALL FUN[X,Y,D]
      DO 3 ]=1,N
      F[]]=F[I]+2.*D[]]
    3 Y[]]=YO[]]+DX*D[]]
      X=X+,5*DX
      CALL FUN]X,Y,D]
      DO 4 I=1,N
      F[]]=.166666666667*]F]][+D[[[]
    4 Y[]]=YO[]]+DX*F[]]
      RETURN
      END
```

```
PROGRAM ALLOCATION

   DUMMY Y        00017 D        00101 YO       00163 F
   00245 ]        DUMMY N        00246 STEP     DUMMY X
   DUMMY DX

SUBPROGRAMS REQUIRED
  FUN

THE END
```

```
C DERIVATIVES FOR THE RADIATING EQUATIONS  D.C. TODD  1-12-67
      SUBROUTINE FUN(X,Y,V)
      DIMENSION Y(3),V(3)
      COMMON D,E,DX,ZF,DS
      X1=-1.+.1*DX
      IF(X1-X)1,3,3
    1 IF(X)2,2,4
    2 X1=X
    3 H=0.
      GO TO 6
    4 H=1.
      X1=1.-.1*DX
      IF(X1-X)6,6,5
    5 X1=X
    6 ZF=Y(3)**4
      V(1)=ZF
      DS=(2.*X*Y(2)+D*((1.+E)*ZF-4.*F(X)*H*X-E))/(1.-X1*X1)
      V(2)=DS
      V(3)=Y(2)
      RETURN
      END

COMMON ALLOCATION

  77776 D          77774 E          77772 DX          77770 ZF
  77766 DS

PROGRAM ALLOCATION

  DUMMY Y          DUMMY V          00007 FUN         00011 X1
  DUMMY X          00013 H
SUBPROGRAMS REQUIRED

  F

THE END
```

```
      FUNCTION F(X)
      F=1.
      RETURN
      END

PROGRAM ALLOCATION

  00002 F
THE END
```

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13. ABSTRACT

This report is a continuation of a study of the effects of internal heat transfer on the temperature of hollow spacecraft and the requirements for thermal modeling. Considered herein is the effect of internal heat transfer by radiation on the temperature distribution. The equation governing the heat transfer of a spherical shell exposed to parallel radiation is derived; conduction and radiation are considered. The general equation is simplified by assuming steady state, and a numerical method is given to solve the steady state equation. A computer program is described which employs the method. Solutions of the steady state equation are graphically presented and discussed. The requirements for temperature preservation in thermal modeling are derived. The possibility of thermal modeling without temperature preservation is discussed. It is observed that for an inside emissivity to outside emissivity ratio greater than one, the requirement for duplication of the other dimensionless ratio can be relaxed.

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1. Shells -- Thermal radiation
2. " -- Heat transfer
3. Thermal modeling
4. Space vehicles - Thermal radiation

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